COMPARING Π^1_2-PROBLEMS IN COMPUTABILITY THEORY AND REVERSE MATHEMATICS

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Reverse mathematics gives us a way to compare the relative strength of theorems by establishing implications and nonimplications over a weak subsystem of second-order arithmetic, typically RCA_0, which corresponds roughly to computable mathematics. In many cases, nonimplications over RCA_0 are proved using ω-models, i.e., models of RCA_0 with standard first-order part. Implication over RCA_0 and over ω-models are not fine enough for some purposes, however, so other notions of computability-theoretic reduction between theorems have been extensively studied. These are particularly well-adapted to a class of theorems that includes a large proportion of those that have been studied in reverse mathematics: A Π^1_2-*problem* is a sentence

$$\forall X\,[\Theta(X) \rightarrow \exists Y\,\Psi(X,Y)]$$

of second-order arithmetic such that Θ and Ψ are arithmetic. The term "problem" reflects a computability-theoretic view that sees such a sentence as a process of finding a suitable Y given an X satisfying certain conditions.

This talk will discuss some of these approaches to studying the relative strength of Π^1_2-problems, focusing in particular on combinatorial examples, including work in [2, 1].

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References

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