

L-MODELS AND R-MODELS FOR LAMBEK CALCULUS ENRICHED WITH ADDITIVES AND THE MULTIPLICATIVE UNIT

ANDRE SCEDROV

Language and relational models, or L -models and R -models, are two natural classes of models for the Lambek calculus. Completeness w.r.t. L -models was proved by Pentus and w.r.t. R -models by Andreka and Mikulas. It is well known that adding both additive conjunction and disjunction together yields incompleteness, because of the distributive law. The product-free Lambek calculus enriched with conjunction only, however, is complete w.r.t. L -models (Buszkowski) as well as R -models (Andreka and Mikulas). The situation with disjunction turns out to be the opposite: we prove that the product-free Lambek calculus enriched with disjunction only is incomplete w.r.t. L -models as well as R -models, in the non-commutative as well as the commutative (linear) case. The derivability problem for the Lambek calculus with conjunction and disjunction is known to be decidable. Adding the explicit multiplicative unit constant changes things drastically. Namely, if we extend Lambek calculus with conjunction by certain simple rules for the multiplicative unit, sound in L -models, then the system becomes undecidable, even in the small fragment with only one implication, conjunction, and unit. In the language with the unit, the algebraic logic of all L -models is strictly included in (does not coincide with) the algebraic logic of regular L -models.

This is joint work with Max Kanovich and Stepan L. Kuznetsov.

UNIVERSITY OF PENNSYLVANIA, PHILADELPHIA, USA
Email address: `scedrov@math.upenn.edu`