AUTOMORPHISM GROUPS OF HOMOGENEOUS STRUCTURES

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Abstract

The relationship between geometric structures and their automorphism groups has been the focus of Klein's *Erlanger Programm* postulated in 1872. In the meantime there has been a wide range of research in the spirit of the program, not just in geometric structures but also with respect to the automorphism groups of other structures. The relationship between structures and their automorphism groups has also given rise to many interesting model theoretic questions.

There are a number of important examples where the automorphism group of a structure and the structure itself carry exactly the same information, in the sense that one can be recovered from the other without any loss of detail. This is, for example, the case for projective spaces and their automorphism groups, but can also be detected in many other settings. In the model theoretic sense this can often be expressed as a bi-interpretation between the automorphism group and the underlying structure, see e.g. [7].

Of particular interest are the automorphism groups of homogeneous structures, which often arise from the model theoretic construction known as the Fraïssé limit. These constructions often lead to ω-categorical structures, i.e. structures which have a unique countable model up to isomorphism. In this case, the connection between the structure and its automorphism group is also reflected in the well-known result due to Coquand stating that two ω-categorical structures are bi-interpretable if and only if their automorphism groups are isomorphic as topological groups, where a basis of the topology for such an automorphism group is given by pointwise stabilizers of finite sets, turning these automorphism groups into polish groups.

This characterization raises the question how difficult it is (in the sense of Borel reducibility) to detect whether two such structures have isomorphic automorphism groups. In joint work with Nies and Schlicht [6] we use the concept of a coarse group to show that the isomorphism relation for oligomorphic subgroups of S_∞ is Borel reducible to a Borel equivalence relation with all classes countable.

In a different direction it can be noted that the automorphism groups of very homogeneous structures are often simple groups or have very few (natural) normal subgroups, see e.g. [5, 8, 9, 1, 3, 2]. From the model theoretic perspective the simplicity of an automorphism group can often be deduced from the existence of a notion of independence, very similar to the one studied in stability theory. While

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many of the structures to which this setting applies are far from stable, the existence of a stationary independence relation often sheds new light on its automorphism group. This is particularly visible in the case of the Urysohn space, or variations thereof, random graphs, etc.

In my talk I will give a survey of some results relating to these questions.

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