LÖB'S PRINCIPLE FOR PAIR THEORIES

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Can we eliminate the various design choices from the statement of the Second Incompleteness Theorem? What makes a coordinate-free version indeed a version of Second Incompleteness Theorem?

There are various approaches to the coordinate-free treatment of the Second Incompleteness Theorem. In this talk, we will zoom in on one such approach. (Some other, but closely related approaches are pursued in [4] and [5].)

Consider a recursively enumerable sequential theory U with full induction for a designated interpretation of number theory N. We can define a big Kripke model (a 'Kripke Universe') $\mathfrak{M}$ with as nodes models of U, such that necessity in this model coincides with arithmetised provability in U relativised to N. (See [2], [3], and [1] for some of the ingredients of the result.) The definition of the accessibility relation of $\mathfrak{M}$ is coordinate-free in the sense that it does not require design choices connected to arithmetisation. We call necessity in $\mathfrak{M}$: *p-validity*. Thus, the equivalence of p-validity with arithmetised provability can be considered as *p-validity elimination*.

If we drop the demand that we have full induction on some interpretation of number theory N, we loose p-validity elimination. So, it would seem that, sadly, the idea of using the big model for a coordinate-free treatment of the Second Incompleteness Theorem goes down the drain. But let's not be hasty. I will argue that, as long as we are aiming at the Second Incompleteness Theorem, there is a modified result that may still count as a coordinate-free treatment. What is more, I will argue that we can view the apparent bug as a feature.

Using an idea of Fedor Pakhomov, we can employ p-validity to prove a version of the Second Incompleteness Theorem for Pair Theories —a place where (full) arithmetisation cannot go. We will sketch an argument that shows that we even get Löb's Principle for non-modal sentences in the case of pair theories.

The full logic of p-validity is currently unknown both for pair theories and for sequential theories.

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References

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