Arithmetical applications of Baaz's generalization method

Lorenzo Sauras-Altuzarra *

Extended abstract

In mathematics, examples are very important, but not all of them are equally good. Intuitively, the more an example reflects the potential of a theorem, the better it is. In fact, if an example sufficiently represents the essence of a theorem, then it can be almost as instructive as the proof itself. Mathematical teaching via examples has many endorsements, notably Babylonian mathematics, which mainly consisted of collections of examples (see Van der Waerden [3, Chapter 3]).

Baaz's generalization method formalizes a way of measuring the quality of an example. Indeed, from a concrete example E of a certain universal theorem T, it generates another universal theorem $t(E)$, with its corresponding proof. A subsequent comparison between T and $t(E)$ may show how well E approximates T.

But there is yet another possible use of this procedure: if applied to an answer to a concrete case of an open problem (such as a proof that 641 divides the fifth Fermat number), it will output a result that can be particularized to a partial answer to the question (such as a sufficient condition for a number to be a divisor of an arbitrary Fermat number).

This talk will explain in detail how to apply this method in elementary number theory. A general explanation of the algorithm is given by Baaz [1].

In addition, if time permits, some arithmetical results (and derived open problems) will be commented, for example:

1. $k \cdot 2^s + 1 \mid F_n$, for every $k, n, r, s \in \mathbb{N}^+$ such that $r \cdot s \leq 2^{n-1}$ and $k \cdot 2^s + 1 \mid k^{2 \cdot r} + 2^{2^n - 2 \cdot r \cdot s}$;
2. $i \mid F_n$, for every $c, i, n \in \mathbb{N}^+$ such that $i \mid (2^{2^{n-1}} - i \cdot c)^2 + 1$; and
3. $2^{2^n - 4 \cdot (n+2)} + i^4 \mid F_n$, for every $i, n \in \mathbb{N}^+$ such that $n > 4$ and $i \cdot 2^{n+2} + 1 = 2^{2^n - 4 \cdot (n+2)} + i^4$

(F_n denotes the n^{th} Fermat number). For proofs of these theorems and some other associated questions, see Sauras-Altuzarra [2].

References

[1] M. Baaz, Note on the generalization of calculations, *Theoret. Comput. Sci.* **224** (1999), 1–2.

[2] L. Sauras-Altuzarra, Some arithmetical problems that are obtained by analyzing proofs and infinite graphs, arXiv:2002.03075 (2020).

[3] B. L. Van der Waerden, *Science Awakening*, Oxford University Press, 1961.

*Institut für Diskrete Mathematik und Geometrie, TU Wien, Wiedner Hauptstrasse 8-10/104, A-1040, Vienna, Austria. E-mail: lorenzo.sauras@tuwien.ac.at.