RELATING STRUCTURE AND POWER: A JUNCTION BETWEEN CATEGORICAL SEMANTICS, MODEL THEORY AND DESCRIPTIVE COMPLEXITY

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There is a remarkable divide in the field of logic in Computer Science, between two distinct strands: one focussing on semantics and compositionality ("Structure"), the other on expressiveness and complexity ("Power"). It is remarkable because these two fundamental aspects are studied using almost disjoint technical languages and methods, by almost disjoint research communities. We believe that bridging this divide is a major issue in Computer Science, and may hold the key to fundamental advances in the field.

In this talk, we describe a novel approach to relating categorical semantics, which exemplifies the first strand, to finite model theory, which exemplifies the second. This was introduced in [1], and substantially extended in [2, 3].

Combinatorial games such as Ehrenfeucht–Fraïssé games, pebble games, and bisimulation games are widely used in finite model theory, constraint satisfaction, modal logic and concurrency theory We show how each of these types of games can be described in terms of an indexed family of comonads on the category of relational structures and homomorphisms. The index k is a resource parameter which bounds the degree of access to the underlying structure. The coKleisli categories for these comonads can be used to give syntax-free characterizations of a wide range of important logical equivalences. Moreover, the coalgebras for these indexed comonads can be used to characterize key combinatorial parameters: tree-depth for the Ehrenfeucht-Fraïssé comonad, tree-width for the pebbling comonad, and synchronization-tree depth for the modal unfolding comonad.

This approach has been extended to guarded fragments [4] and generalized quantifiers [5]. Applications to homomorphism preservation theorems are described in [6, 7], and to Lovász-type theorems on isomorphisms of relational structures in [8]. An axiomatic framework in terms of *arboreal covers* of extensional categories is developed in [9].

References

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