DEFINABILITY IN THE TURING DEGREE STRUCTURES

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Among the most difficult and valuable problems of the computability theory definability problems occupy the most significant place.

The most important achievements in this field include the proof of the definability of the jump operator in the global Turing degree theory by Shore and Slaman [5], of the set of computably enumerable (c. e.) degrees $\mathcal{R}$ in the local Turing degree theory ($D(\leq_T \emptyset')$) by Slaman and Woodin [6], the existence of an infinite definable set of c. e. degrees in the finite levels of the Ershov difference hierarchy by Arslanov, Kalimullin and Lempp [1], the proof of the definability of the e-jump in the enumeration degrees by Kalimullin [3], the definability of the all jump classes Low_n and $\mathrm{High}_{n-1} (n \geq 2)$ in the $\mathcal{R}$ degrees by Nies, Shore and Slaman [4], the definability of the total enumeration (e-) degrees by Cai, Ganchev, Lempp, Miller and Soskova [2].

In recent years, an intensive search for the "natural" definition for the jump operator, in particular for the degree $\mathbf{0}'$ in the global Turing degree theory, a search for natural definitions for classes of c. e. degrees, for jump classes Low_n and High_n, for the degree classes within different levels of the Ershov hierarchy in the local Turing degree theory was carried out. These studies have produced a number of encouraging results.

These problems are closely related to some other major open problems of computability theory, such as the existence of nontrivial automorphisms of structures of degrees of unsolvability.

In my talk I will provide an overview of these studies.

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