Gentzenizing R

Mirjana Ilić*

University of Belgrade, Faculty of Economics, Belgrade, Serbia
mirjanailic@ekof.bg.ac.rs

Abstract

It is well–known that extending a sequent calculus of positive relevant logic, for example Dunn's LK_+ [4], so as to handle a negation is not trivial. Belnap [1], solved this problem, using a concept of 'Display logic', but by going outside the standard vocabulary for R. Namely, to the standard $\{\rightarrow, \wedge, \vee, \sim\}$ he added not only, t and $\circ$, which are also needed in LK_+, but T and $\sim_b$, where T is the disjunction of all propositions and $\sim_b$ is Boolean negation. Another solution of this problem was presented by Brady [3], who in addition to t and $\circ$, used also the classical negation, denoted by $-$, and additional structural connective $\star$, corresponding to $\otimes$, defined by $\alpha \otimes \beta = \alpha \wedge - \sim \beta$, in order to set up the left–handed sequent system with signed formulae, for R. Significantly simpler sequent calculus was presented by Bimbó and Dunn [2], but only for the fragment $R^t_{\rightarrow}$ of R.

We have tried to set up a sequent system for R, less entangled than Brady's or Belnap's. Bearing in mind that RW allows a simple gentzenization on the standard vocabulary, GRW [6], we formulate the system GR by adding the intensional contraction rule

$$\frac{\vdash \Gamma[\Pi; \Pi]}{\Gamma[\Pi]} \text{ (WI)}$$

to GRW. We prove that GR presents the sequent calculus for R. Unfortunately, the rule of cut cannot be eliminated in GR [7].

References

[1] N. D. Belnap, Jr., *Display Logic,* Journal of Philosophical Logic, **11** (1982), pp. 375–417.

[2] K. Bimbó, J, M. Dunn, *New consecution calculi for $R^t_{\rightarrow}$,* Notre Dame Journal of Formal Logic, **53** (2012), pp. 491–501.

[3] R. T. Brady, *Gentzenizations of relevant logics with distribution*, The Journal of Symbolic Logic, **61(2)** (1996), pp. 402–420.

[4] J. M. Dunn, *A 'Gentzen system' for positive relevant implication*, The Journal of Symbolic Logic, **38** (1973), pp. 356–357.

[5] G. Gentzen, *Investigations into logical deduction*, The Collected Papers of Gerhard Gentzen, Szabo, M. E. (ed.) North–Holland (1969), pp. 68–131.

[6] M. Ilić, B. Boričić, *A cut-free sequent calculus for relevant logic RW*, Logic Journal of IGPL, **22(4)** (2014), pp. 673-695.

[7] M. Ilić, B. Boričić, *A note on the system GRW with the intensional contraction rule*, to appear in Logic Journal of IGPL, DOI:10,1093/jigpal/jzaa002.

*This work is supported by the Ministry of Science and Technology of Serbia, grant number ON174026.