A Labelled Sequent Calculus for HYPE

Abstract

In this work, we will present and discuss a calculus for HYPE's system as developed in [Lei18]. G3HYPE is a labelled calculus built to provide a proof system for HYPE's model theory, as it allows us to express in syntactic terms statements concerning not only formulas in the language, but possible intercurrent relations between different states as well. Structural rule admissibility will be shown to hold for the system, along with some of the most important metatheorems. Possible recovery of different logics in the system will be finally shown.

This work is devoted to finding a proper sequent-style calculus reflecting most of the attractive features that HYPE semantics display by the use of modern proof theoretic techniques.

HYPE is a system of non-classical logic developed in [Lei18]. Philosophical and technical motivation for the system are manifold and will not be discussed in length here. They include, and are not limited to, the study of an easily extendible semantic framework useful for specifying different logics, for applications in the field of semantic paradoxes, and for a possible background system for hyperintensional operators. While some of this research has already been hinted at or partially developed in [Lei18], other works are, at the time of writing, in development.

Our aim however will not only consist in offering a completion of [Lei18] from a proof theoretical side. We will in fact try to provide and argue for new syntactic grounds on which a study of HYPE's characteristic features can be brought on.

Taken as a logic per se, HYPE's official one,[1] is not difficult to spell out starting from the axioms provided in [Lei18]. This system, while possessing the advantage of being simple to define, seems to be not expressive enough to represent the possibility provided by such a rich semantics, which includes relations typical of mereological systems (fusion), combined with star states and (in)compatibility relations. Moreover, the strategy adopted for proving (strong) completeness through the construction of a canonical model is particularly involving. As we shall see, it seems to lead to an inexact result, namely the fact that the axiomatic system presented in [Lei18] is complete for the variable domain variant of HYPE.[2] For these reasons, we will introduce a different kind of framework, namely a linguistic extension of a G3-style classical sequent calculus. With such system we aim at providing a Classical logic base calculus for HYPE's characteristic semantic clauses and model-theoretic relations by representing them in the language of the derivation. To this end, a system of labels in the style of [Neg05] and [DN12] is employed. Similarly to labelled calculi in Modal and Intuitionistic logics, variable and constant labels will be used to label formulas.

This permits us to obtain a calculus G3HYPE with an algorithmic Cut admissibility procedure. In this case, the metatheorems will be proved in a simple and direct way in order to show such closeness to the actual semantics from one side and the benefit of the employed proof theoretic machinery from the other. The completeness proof will indeed be shown by the simple construction of a proof-search reduction tree, by generalising the method of Schütte-Takeuti [Tak13]. By the internalisation of HYPE's first-order semantics, however, we can actually achieve more, namely, we will obtain a system that enables us to reason with HYPE's model

[1] There are many other ways in which HYPE semantics is sharpened. We are going to consider the ones specified in [Lei18] for the propositional fragment

[2] In order to obtain the constant domain one, substitution of logical equivalents must be assumed in the axiomatic system, as it is not a derivable property of it.

theory, and in which many semantic observations made in [Lei18] can be derived in the system without ad-hoc additions.

Finally, we would like to remark that the logics recaptured by extensions of HYPE by imposing restriction on the models can therefore be recaptured in the proof system by formalising such restriction as rules over the relations between the labels. This logics include Classical logic, Intuitionistic one, First Degree Entailment (FDE) [Bel77], Strong Kleene logic (K3) [Kle38], Logic of Paradox (LP) [Pri79] and therefore, as we will show, Strict-Tolerant logic (ST), that is, Classical logic minus Cut [CERvR12].

References

[Bel77] Nuel D Belnap. A useful four-valued logic. In *Modern uses of multiple-valued logic*, pages 5–37. Springer, 1977.

[CERvR12] Pablo Cobreros, Paul Egré, David Ripley, and Robert van Rooij. Tolerant, classical, strict. *Journal of Philosophical Logic*, 41(2):347–385, 2012.

[DN12] Roy Dyckhoff and Sara Negri. Proof analysis in intermediate logics. *Archive for Mathematical Logic*, 51(1-2):71–92, 2012.

[Kle38] Stephen Cole Kleene. On notation for ordinal numbers. *The Journal of Symbolic Logic*, 3(4):150–155, 1938.

[Lei18] Hannes Leitgeb. HYPE: A system of hyperintensional logic (with an application to semantic paradoxes). *Journal of Philosophical Logic*, 2018.

[Neg05] Sara Negri. Proof analysis in modal logic. *Journal of Philosophical Logic*, 34(5-6):507, 2005.

[Pri79] Graham Priest. The logic of paradox. *Journal of Philosophical logic*, 8(1):219–241, 1979.

[Tak13] Gaisi Takeuti. *Proof theory*. Dover, 2013.